Realization of the Exactly Solvable Kitaev Honeycomb Lattice Model in a Spin Rotation Invariant System

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The exactly solvable Kitaev honeycomb lattice model is realized as the low energy effect Hamiltonian of a spin-1/2 model with spin rotation and time-reversal symmetry. The mapping to low energy effective Hamiltonian is exact, without truncation errors in traditional perturbation series expansions. This model consists of a honeycomb lattice of clusters of four spin-1/2 moments, and contains short-range interactions up to six-spin(or eight-spin) terms. The spin in the Kitaev model is represented not as these spin-1/2 moments, but as pseudo-spin of the two-dimensional spin singlet sector of the four antiferromagnetically coupled spin-1/2 moments within each cluster. Spin correlations in the Kitaev model are mapped to dimer correlations or spin-chirality correlations in this model. This exact construction is quite general and can be used to make other interesting spin-1/2 models from spin rotation invariant Hamiltonians. We discuss two possible routes to generate the high order spin interactions from more natural couplings, which involves perturbative expansions thus breaks the exact mapping, although in a controlled manner.

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I. INTRODUCTION.

Kitaev's exactly solvable spin-1/2 honeycomb lattice model[1] (noted as the Kitaev model hereafter) has inspired great interest since its debut, due to its exact solvability, fractionalized excitations, and the potential to realize non-Abelian anyons. The model simply reads

$$H_{\text{Kitaev}} = -\sum_{x-\text{links } <jk>} J_x \tau_j^x \tau_k^x - \sum_{y-\text{links } <jk>} J_y \tau_j^y \tau_k^y - \sum_{z-\text{links } <jk>} J_z \tau_j^z \tau_k^z \tag{1}$$

where $\tau^{x,y,z}$ are Pauli matrices, and x, y, z-links are defined in FIG. 1. It was shown by Kitaev[1] that this spin-1/2 model can be mapped to a model with one Majorana fermion per site coupled to Ising gauge fields on the links. And as the Ising gauge flux has no fluctuation, the model can be regarded as, under each gauge flux configuration, a free Majorana fermion problem. The ground state is achieved in the sector of zero gauge flux through each hexagon. The Majorana fermions in this sector have Dirac-like gapless dispersion resembling that of graphene, as long as $|J_x|$, $|J_y|$, and $|J_z|$ satisfy the triangular relation, sum of any two of them is greater than the third one[1]. It was further proposed by Kitaev[1] that opening of fermion gap by magnetic field can give the Ising vortices non-Abelian anyonic statistics, because the Ising vortex will carry a zero-energy Majorana mode, although magnetic field destroys the exact solvability.

Great efforts have been invested to better understand the properties of the Kitaev model. For example, several groups have pointed out that the fractionalized Majorana fermion excitations may be understood from the more familiar Jordan-Wigner transformation of 1D spin systems[2,3]. The analogy between the non-Abelian Ising vortices and vortices in $p + ip$ superconductors has been raised in serveral works[4–7]. Exact diagonalization has been used to study the Kitaev model on small lattices[8]. And perturbative expansion methods have been developed to study the gapped phases of the Kitaev-type models[9].

Many generalizations of the Kitaev model have been